... I do and I understand, and then I forget. The role of memory in Mathematics Education

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Memory has received very little attention in mathematics education literature in recent years, except in the negative connotation of rote learning. However since memory and cognition are very closely linked memory has an important part to play in many, if not most aspects of learning mathematics. The relationships between memory and understanding, memory and assessment, memory and culture, and memory and problem solving are discussed and a case is made for a much more positive role for memory in mathematics education research.

Introduction

It seems to me that the task of a keynote speaker differs from that of other presenters at a conference like this. For those presenting research papers the task is to discuss their paper with an audience who have indicated their interest in the topic and the focus is principally on the written paper. For a keynote speaker, however, the emphasis is on the oral presentation, and the challenge is to provoke, interest, and, to some extent, entertain a large group of people with differing backgrounds and interests. Not unlike teaching mathematics classes really! In almost forty years of teaching mathematics in a variety of institutions, I have never written a mathematics lesson or lecture and for this reason I have found writing this paper for publication several months before I will speak on the topic a difficult and unsatisfactory process. I will certainly not read this paper at the conference, and reserve the right to change the contents significantly both before, and during, the address. However, I understand that I must not change the topic altogether and I will try to remember that.

The human memory is fascinating and amazing. Why is it that of all the millions of stimuli which we receive each day we remember some and forget the others? Why is it that if I wish to recall someone's name I very seldom can at the time, but if I have tried, the name will pop into my head much later, often at night and always long after the information is any use to me? I have much sympathy with the Russian poet Joseph Brodsky who wrote: "More than anything, memory resembles a library in alphabetical disorder, and with no collected works by anyone".

In my, and I suspect most of your, experiences of learning mathematics at school, memory played an important and explicit part. We were sent away to learn things for homework and were tested the next day. We were given seemingly endless lists of practice exercises so that procedures became semi-automatic. I am certainly not intending to argue that this was appropriate. We now know a good deal more about the way in which people learn mathematics. However, I do propose to argue that, in our anxiety to change this approach to learning we are in danger of underestimating the role which, in my view, memory must have in mathematics education.

In recent years, consideration of the role of memory in the learning of mathematics has been almost absent in mathematics education research and literature. There is, of course, a significant body of psychological research into memory, focussing principally on laboratory studies of semantic and episodic memory (Bjorklund, 1995). Little of this seems to have influenced mathematics education research, probably because of its lack of direct relevance to the mathematics classroom. Very little recent mathematics education research mentions memory at all. In preparing this paper I reviewed the past five years entries in Zentralblatt fur Didactic der Mathematik and found just six papers out of the hundreds reviewed which addressed the topic. Most of these related to learning disabled students. I looked in the indices of all of the recently published books on Mathematics Education on the shelves of the library of my University and none of them had a reference to memory. The abstracts of the papers published in the proceedings of last year's MERGA conference (Clarkson, 1996) also contained no reference to memory at all. Finally, I looked

through the Proceedings of ICME 7 (Gaulin *et al*, 1992) and found just two references which in both cases were negative, referring to the contrast between memorisation and other, more desirable, aspects of mathematics learning.

For example, Silver (1992, p 375) wrote "... mathematics (should) be taught through activities that invite students to think, reason, explain, and justify, rather than simply to memorise and imitate". I would, of course, have no quarrel with this statement at all. However, it is clear to me that memory does have a part to play in thinking, in reasoning, in explaining, and in justifying. In our anxiety to avoid the dangers of rote learning, we have, I believe, forgotten the essential role which memory plays in all cognitive activity.

Because of this critical relationship between memory and cognition, I believe that memory is an important aspect of many, if not most, facets of matheamtics education. For example, the initial relationship to constructivism is clear; children construct their own memories (Piaget and Inhelder) and we might well be interested in activities which are related to the construction process. There is also an obvious connection with assessment issues. It is in this context that much of the negative connotation of memorisation occurs. There are also, I believe, relationships to the general concepts of knowledge and learning, to metacognition, to motivation, to issues of culture and gender, and to problem solving and investigations. In this presentation, I would like to explore just one or two of these relationships and to make a general case for a more significant consideration of memory issues in mathematics education research.

Memory and Understanding

Understanding has been a major topic in the mathematics education literature for more than twenty years, much of the discussion having its origins in the work by Skemp (1976) on instrumental and relational understanding. The latest contribution is the substantial book by Anna Sierpinska (1994) entitled 'Understanding in Mathematics'. From the point of view of this presentation, it is significant that in the index of the book there is no reference to memory at all. Within the book itself the only mention I could find of memory were two rather dismissive references (p 7 and 70) to a cognitive science approach to understanding. Sierpinska writes:

> Among the many views of understanding, there is one which identifies an act of understanding with a retrieval of a 'frame' or 'script' from memory, sometimes called the 'computer metaphor approach'. (p 7)

> There are several deceiving aspects of this approach. One is that it represents the functioning of the human mind as mechanical, automatical... (p 70)

There is clearly more to understanding than retrieval, but it is this classification of anything to do with memory as mechanical and automatic, and consequently undesirable, which is at the heart of the rejection of memory as an important aspect of mathematics education.

It seems to me that implicit in much of the recent mathematics education literature are the following assumption concerning understanding:

- Understanding is permanent. Once you have understood something you will always understand it.
- If you understand something you will be able to remember it.

My own experience of learning mathematics and my observations of students learning mathematics at many levels lead me to reject both of these assumptions.

As an undergraduate student I learnt a great deal of mathematics and understood at least some of it quite well. A large part of that mathematics I have not used for more than forty years and it is now well and truly forgotten and, given any sensible definition of understanding, I certainly no longer understand it. Understanding is not permanent. There is no contradiction, in my view, in saying "I used to understand that". There have also been many times when I have been engaged in a mathematical task and I have been

completely unable to recall a fact, formula or procedure which I both understood when I learnt it and understand when I am reminded. Understanding is no guarantee of recall.

The same applies to my students. One of the most frustrating features of helping individual students with their problems arises not with students who have misunderstood the concept or procedure involved, at least then you have something to work on, but with those who show that they do understand the right procedure but continue to use a previously learned wrong procedure.

What then is the relationship between memory and understanding? Do we remember things differently if we understand them? Is there a step which we should be encouraging students to take after they understand so that they will remember? Are there activities which would promote both understanding and memory? All of these seem to me to be the basis for good research questions.

Memory and assessment

The relationship between memory and traditional forms of assessment by examination is obvious. It is the excesses of rote learning, imitation, and regurgitation associated with examinations which has led to the rejection of memorising as a valid activity in mathematics education. However, it is not possible to remove memory from assessment tasks altogether, even if this were seen as desirable.

Firstly, there are some features of mathematics which are matters of symbolism or definition which have to be remembered rather than understood. One simply has to learn that 5!! is the symbol for the product 5x3x1. There is no logic, or understanding, involved. A logical response would be extreme surprise that it was a five. The number words and symbols themselves are in this same category. How do we learn that the word eight or the symbol 8 is associated with certain sets of objects if not by rote? Having learnt this we can, of course, understand that, and why, 8 comes between 7 and 9. If this is the case it seems to me to be essential to assess whether or not the student can remember what the symbol 8 represents.The testing of recall is entirely appropriate in some circumstances.

Secondly, in order to reduce the emphasis on memory in traditional mathematics examinations, it has become common to provide information for students to use during the examination. This may range from a list of formulae to a completely open book examination where students may take in any reference material they wish. Do we really understand the effect of this on student learning? I have certainly not seen any research in this area. I do however have colleagues whose opinion I respect who are concerned that the message we are giving to students is that it is not necessary, and is perhaps even undesirable, to commit formulae and procedures to memory. My colleagues feel that this has a significant adverse effect on future learning. Are they right?

Thirdly, if we abandon traditional forms of assessment, there is still an important memory component. Ken Carr (1994) suggests interviewing, observation, student portfolios, self assessment, and cooperative assessment as alternative assessment procedures for teachers. At least three of these have important links to memory. In interviewing, or observing, a student one immediately becomes aware of what the student can remember. In self assessment, with its obvious links with metacognition, one would hope that the students themselves would become aware of what they can remember and to be able to reflect on that. The area of metamemory (Bjorkland, 1995, p 248), for knowledge of the workings of one's own memory, seems to me to be well worth researching in mathematics education.

Memory and Culture

There are significant differences in the value which cultures place on memory. For example, since Maori culture was originally oral, without a written language, the ability to remember and repeat information was essential to the preservation of the culture and was consequently highly valued. The ability to recite one's whakapapa, or geneology, for many generations and to be able to link oneself to others in this way remains an important feature of Maori culture. Many Asian cultures also seem to place a

high value on memory, at least as far as education is concerned. Asian students are often characterised as rote learners. Wong (1996), in discussing the performance of Asian students, states that "lessons are structured as one continuous practice session for public examinations" and suggests that the origin of this examination culture lies in Confucian values and view of life.

What do we say to the Maori students and the significant, and growing, number of Asian students in New Zealand? Your values are wrong and we know best? All the lessons of Maori education for the past one hundred and fifty years indicate the futility and immorality of this approach. Do we need to reject their skills of practice and memorisation? Could we not, as Wong suggests, "Explore the Asian style of understanding through repetitive learning"? Again there is ample scope for research.

Memory and problem solving

The bad name which memory has in mathematics education is also to be found in writings on problem solving. Thomas and Kota (1996) writing on problem solving in algebra write:

> Brown et al (1988) expressed the opinion that students appear to cover their inability to translate the relationship between the words into symbolic form or to solve the resulting equations resorting to memorizing rules and procedures that they eventually come to believe represent the essence of algebra. (p 564)

They are right, of course. Memorization of rules and procedures is not the essence of algebra. However it is equally true that if we cannot remember any algebraic rules or procedures we are, it seems, to me most unlikely to be able to identify the relationship between the words of the statement of a problem and a symbolic form. Have we again thrown out the baby with the bath water?

What is the relationship between problem solving ability and memory? Certainly, memorization is no guarantee of problem solving ability, but without some recall we cannot begin to solve problems. Is there a qualitative difference between the recall ability of good and poor problem solvers? Are there activities which we could devise which would help students in this area?

Summary

I could continue and consider memory and motivation, memory and gender issues, etc., but I hope I have made what is a simple, but important, point. In recent research in mathematics education we have forgotten to include memory as a factor except in a negative way. Perhaps we need some new terms to enable us to distinguish between the positive and negative aspects of memory. A term for 'memory with understanding' might be helpful, and another to contrast 'memorization' (with its connotations of rote learning) with the more positive aim of using memory to provide access to our understanding.

In his book 'Forty year's on' (1969), Alan Bennet describes how Rumpole's teacher proclaimed that: "Education is what is left after you have forgotten all that you have ever learnt". The teacher went on to suggest that Rumpole was seeking to circumvent this process by learning as little as possible. Perhaps we, like Rumple, are also in danger of circumventing the process by not expecting our students to remember anything so that they have nothing to forget.

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